Effect of energy scale imperfections on results of neutrino mass measurements from β-decay

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Abstract

The effect of undetected imperfections of the energy scale on the results of electron spectroscopy experiment is studied for the conditions close to the tritium β-decay experiment KATRIN [1]. A numerical model of the experiment is developed, verified, and used to deduce general trends of fitted parameters. The effects of energy bias and of wrong slope of the calibration line are studied in more detail.

Key words: electron spectroscopy; tritium decay; energy calibration; simulations;

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1 Introduction

In this work we study the effect of possible energy scale imperfections on the result of the tritium β-decay experiment KATRIN [1]. This is a new generation experiment the aim of which is to reach the sensitivity to the electron antineutrino mass in order of 10^{-1} eV. (Note that the present upper limits of this mass are 2.2 eV [2] and 2.5 eV [3], both at the 95% C.L.) The essence of the experiment is the measurement of tritium β–spectrum by means of a new giant integrating electrostatic electron spectrometer [1].

The energy region measured will be a short interval near the endpoint which is $\sim$18.6 keV. This means that the retarding voltage of the order of 10^4 V must be kept as constant as possible. The long-termed precision of present high-voltage power supplies as well as of the corresponding voltage dividers is several *ppm* at best. There exist methods (see e.g. [4–6]) how to detect and/or eliminate some kinds of energy scale instabilities.

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However there always remain tiny instabilities which one is unable to register with the present measuring instruments. We devote this work to possible effects of such undetected instabilities on the derived value of the neutrino mass. The methods described here may be useful in planning many spectroscopy experiments where calibration and/or stability of the scale are of topical importance for correctness of the result.

2 Basic concept

The tritium β–spectrum is given by the formula (e.g. [7])

$$\begin{aligned}\beta(E,Q,m_\nu) = N_s\, F(Z,E)\, \sqrt{E(E+2m_ec^2)}\left(E+m_ec^2\right) \times \\ \sum_i \Big[\, \omega_i\, (Q-W_i-E)\, \sqrt{(Q-W_i-E)^2 - m_\nu^2c^4}\ \times \\ \Theta(Q-W_i-E-m_\nu c^2)\Big]\,,\end{aligned} \tag{1}$$

where N_s is a norm of the spectrum, E is the electron kinetic energy, Q is the maximum kinetic energy of electron assuming massless neutrino, m_e and m_ν are electron and neutrino masses, respectively. W_i stands for the energy of the ith excited level (rotational-vibrational or electronic state excitation) of the daughter $(^3\mathrm{HeT})^+$ molecule and ω_i is probability of the transition to that level [8]. The step function Θ guarantees the energy conservation.

The Fermi function, $F(Z,E)$, is defined e.g. in [9] but for our purpose it can be approximated with sufficient precision [10] by

$$F(Z,E) = \frac{x}{1-\exp(-x)}\left(a_0 + a_1\frac{v_e}{c}\right), \quad x = \frac{2\pi Z\alpha c}{v_e}. \tag{2}$$

Here, Z is atomic number of the daughter nucleus ($Z=2$ in our case), α is the fine structure constant, v_e is the electron velocity, and $a_0 = 1.002037$ and $a_1 = -0.001427$ are empirical constants.

As stated above, the spectrum will be measured by an integrating spectrometer. Then the spectrum recorded will be

$$S(T; N_s, N_b, Q, m_\nu) = N_s \int_0^\infty \beta(E,Q,m_\nu)\, R'(E,T)\, \mathrm{d}E + N_b, \tag{3}$$

where $\beta(E,Q,m_\nu)$ is the differential spectrum given by Eq.(1), N_s is the normalizing factor ("amplitude"), N_b is background, $R'(E,T)$ is a spectrometer response function,

and T denotes the energy determined by spectrometer retarding voltage. The response function here includes the corrections for electron scattering within the tritium source

$$R'(E,T) = \int_0^{E-T} R(E-\varepsilon,T) \times [P_0\delta(\varepsilon) + P_1 f(\varepsilon) + P_2(f \otimes f)(\varepsilon) + \ldots]\, \mathrm{d}\varepsilon, \tag{4}$$

where R is the theoretical spectrometer transmission function [1], f is the energy differential cross section [11], P_i are probabilities of an electron to be scattered i times [12], δ represents the Dirac δ-function and $\otimes$ symbol denotes convolution. The function R depends on the spectrometer setup, in particular on magnetic fields B_s in the source region and B_A in the analyzing plane. The response function for the setup typical for the KATRIN experiment is presented in Fig.1.

To grub the relevant information out from the measured spectra, the least–squares–method (LSQ) is usually utilized. We minimize the functional

$$\chi^2(N_s, N_b, Q, m_\nu) = \sum_i \left(\frac{S_{exp}(T_i) - S(T_i; N_s, N_b, Q, m_\nu)}{\sigma(T_i)} \right)^2 \tag{5}$$

with respect to the four fitted parameters – N_s, N_b, Q, and m_ν. S_{exp} denotes the experimental spectrum, S is the theoretical one, Eq.(3), and σ stands for experimental standard deviation. Sum runs over all measured points.

The standard deviations of the fitted parameters are usually derived from diagonal elements of the covariance matrix which is defined as

$$\mathbf{C}^{-1}_{kl} = \sum_i \frac{\partial S(T_i;\tilde{x})}{\partial x_k} \frac{\partial S(T_i;\tilde{x})}{\partial x_l} \frac{1}{\sigma^2(T_i)}, \tag{6}$$

where $\tilde{x}$ denotes the set of the fitted parameters which minimizes χ^2, Eq.(5). As we will show below, this approach is inapplicable in our case and we must use another method.

3 Numerical model

The aim of the experiment is to determine the value of the neutrino mass, m_ν. In fact, we must use the experimental spectrum to determine four unknown parameters, in particular m_ν, Q, N_s, and N_b. In order to see how various imperfections affect the results we apply the following procedure. We repeatedly simulate experimental spectra with slightly deformed energy scale, evaluate them and compare the results.

In our study, we use the values of various constants (as, e.g., those in the energy differential cross section f etc.) typical for the experiment KATRIN. They can be found in

[1,11,12]. For illustration, we present here only $N_s \approx 8.9 \times 10^{-14}$ s^{-1}eV^{-5} and $N_b \approx 10^{-2}$ s^{-1}.

At present, it is known that the neutrino mass, m_ν, is close to zero. This implies a trouble – if repeating the experiment several times, we sometimes get $m_\nu < 0$ (purely from statistical reasons). The formula (1), however, loses its meaning for $m_\nu < 0$ and we must find a suitable analytical continuation. A reasonable choice has to satisfy the following two conditions [12]:

(1) to let χ^2 (Eq.(5)) symmetric as a function of m_ν (with respect to zero),
(2) to be as smooth as possible.

Both these conditions are fulfilled in a good approximation when redefining the differential spectrum as

$$\beta(E,Q,m_\nu) = \begin{cases} \text{Eq.(1)} & \text{for } m_\nu \geq 0, \\ \beta'(E,Q,m_\nu) & \text{for } m_\nu < 0, \end{cases} \tag{7}$$

where, for $m_\nu < 0$,

$$\begin{aligned} \beta'(E,Q,m_\nu) = {} & N_s\, F(Z,E) \sqrt{E(E+2m_e c^2)} \left(E + m_e c^2\right) \times \\ & \sum_i \Big[\omega_i \left(Q - W_i - E\right) \sqrt{(Q-W_i-E)^2 + m_\nu^2 c^4} \;\times \\ & \Theta(Q - W_i - E) \Big] . \end{aligned} \tag{8}$$

The above formulae are used when fitting m_ν. In the neutrino community, there is usually used m_ν^2 as a fitted parameter. Here we only state that when fitting m_ν^2, we may also use Eq.(7) but now $\beta'(E,Q,m_\nu^2 < 0)$ is given by Eq.(1) with change $\Theta(Q - W_i - E - m_\nu c^2)$ to $\Theta(Q - W_i - E)$.

Now, we can start the simulations. We create spectra following Eq.(3) with the β-function of Eq.(7) instead of Eq.(1). We evaluate those spectra by minimizing χ^2, Eq.(5) and search for tendencies in the resulting m_ν. For the minimizing, we use a combination of simplex, Levenberg-Marquardt, and Gauss-Newton methods. For the determination of the standard deviations of the fitted parameters, we do not use the usual method with the covariance matrix, Eq.(6), since $\lim_{Q-E\to m_\nu +} \frac{\partial\beta(E,Q,m_\nu)}{\partial Q} = +\infty$ and similarly for the derivative in m_ν. Therefore, we use following two methods to get the standard deviation estimations:

Independent measurements method: A number (1000 typically) of spectra is simulated and evaluated. The derived minimizers are supposed to be a set of independent realizations of a random variable. The covariance matrix is then calculated using its definition only. The method is rough and can not be used to give a covariance matrix estimation

in the case of the real experimental data. In order to describe in more detail some features of fitted parameter deviations, e.g. asymmetry, the next method was used.

Standard error ellipse [13]: We find minimizers, $\tilde{x}$, of χ^2 for the spectrum studied ($\chi^2(\tilde{x}) = \chi^2_0 = \text{min.}$). Then the set of x fulfilling the condition $\chi^2(x) = \chi^2_0 + 1$ determines the wanted standard deviations. (For the details of the method implementation see [12].)

The standard error ellipse method can be used to analyze each specific case separately, as well as it can be applied to unrandomized spectra, in order to study systematics. However the method, as we implemented it, provides no information about correlation coefficients of the fitted parameters. On the other hand, the method respects a possible asymmetry of χ^2. We consider this fact to be *the main advantage* of the standard error ellipse method.

In order to check both the model and the corresponding software, theoretical unrandomized spectra were evaluated for a broad region of input parameters. Differencies between the initial (creating) and the final (fitted) neutrino mass were observed. The worst value was less than 10^{-5} eV, typical value was less than 10^{-7} eV; restricting ourselves to the massless neutrino, typical values were below 10^{-8} eV.

In the following – until stated otherwise explicitly – we simulated spectra for energies T_i from 18 545 to 18 577 eV with the step of 0.5 eV, Q=18 575 eV, and corresponding to the measurement time of one year.

4 Search for m_ν or m_ν^2?

The question may arise whether to work with linear or squared neutrino mass, i.e. whether to fit m_ν or m_ν^2. Generally, people prefer using m_ν^2 since it appears in the spectrum, Eq.(1), directly. On the other hand, what we want as a result is m_ν. We have developed a software which can manage both cases to illuminate that problem.

From the mathematical point of view, there is no difference between whether we fit m_ν or m_ν^2. What must be verified is whether their derived standard deviations are mutually compatible. If we present a physical quantity as $\bar{x}\,^{+\sigma_x^+}_{-\sigma_x^-}$, then our deviations should fulfill

$$\sigma^\pm_{m_\nu^2} = 2m_\nu\sigma^\pm_{m_\nu} \pm \left(\sigma^\pm_{m_\nu}\right)^2 . \tag{9}$$

(Note that the word "physical" means that we restrict ourselves here to positive both m_ν and m_ν^2.) We must verify whether the squared-mass deviations derived directly conform those determined by Eq.(9) from the derived linear-mass deviations. To this aim, we evaluated simulated unrandomized spectra (m_ν from 0 to 0.5 eV) by the standard error ellipse method using both algorithms – fitting m_ν and m_ν^2. Comparison of deviations obtained (Table 1) confirms compatibility of both approaches.

Table 1
The 'upper' deviations σ^+ as derived by direct fitting of m_ν – second column, direct fitting of m_ν^2 – third column, and from Eq.(9) – fourth column, respectively. The quantity δ is difference between the both $\sigma^+_{m_\nu^2}$'s.

m_ν [eV]	$\sigma^+_{m_\nu}$ [eV]	$\sigma^+_{m_\nu^2}$ der. [eV^2]	$\sigma^+_{m_\nu^2}$ fit. [eV^2]	δ [eV^2]
0.0	0.205 973 6	0.042 425 1	0.042 425 1	$<10^{-7}$
0.1	0.129 050 5	0.042 464 1	0.042 464 1	$<10^{-7}$
0.2	0.087 320 1	0.042 552 8	0.042 552 8	$<10^{-7}$
0.3	0.064 318 5	0.042 728 0	0.042 728 0	$<10^{-7}$
0.4	0.050 503 0	0.042 952 9	0.042 952 9	$<10^{-7}$
0.5	0.041 525 8	0.043 250 2	0.043 250 2	$<10^{-7}$

The next feature which our model must satisfy is the correct dependence of the determined deviation on the measurement time. We created simulated spectra as above (this time for m_ν=0 only) and evaluated them by the standard error ellipse method again. We found that the time dependence fulfills

$$\frac{t}{t'} = \left(\frac{\sigma'_{m_\nu}}{\sigma_{m_\nu}}\right)^4 = \left(\frac{\sigma'_{m_\nu^2}}{\sigma_{m_\nu^2}}\right)^2 , \tag{10}$$

which shows that – from statistical point of view – m_ν^2 is a more suitable fitted parameter than m_ν is.

Another important question is the dependence of the mass standard deviation on the value of the mass itself. We have studied it in the same manner as above – creating the simulated spectra for various neutrino masses and evaluating them by the standard error ellipse method. The results for $\sigma^+_{m_\nu}$ are shown in Tab. 1. It turned out that $\sigma_{m_\nu^2}$ is within 2 % independent of the neutrino mass which conforms with Eq.(9).

In the following we present only results concerning *squared* mass. Nevertheless we performed all the calculations for linear mass as well and proved that in all cases the Eq.(9) is fulfilled very well.

5 Determination of confidence levels

To connect the determined standard deviations of m_ν and/or m_ν^2 with confidence levels we create the neutrino mass cumulative distribution function by integrating the neutrino mass histogram.

We evaluated 2500 spectra created to correspond to the expected KATRIN experimental conditions and the mass m_ν=0. The histograms are illustrated in Fig. 2 for both

cases – fitted m_ν as well as m_ν^2. The surprising function form of the neutrino linear mass density may be explained as follows: Let $H(m_\nu^2)$ be the neutrino mass squared density. In the physical region (i.e. m_ν, $m_\nu^2 \geq 0$) the linear mass density may be approximated as

$$G(m_\nu) = H(m_\nu^2) \times |2m_\nu|. \tag{11}$$

From Eq.(11) it is clear that despite of the nonzero value of the mass squared density, $H(m_\nu^2)$, the linear mass density vanishes for m_ν=0.

Assume – as an example – that we measured a spectrum under the conditions for which the density function had been calculated. Further, assume that the spectrum evaluation resulted in m_ν=0 and the value of $\sigma^+_{m_\nu}$ is 0.206. Then we can conclude that the upper bound for the neutrino mass is 0.206 eV at 65.8% CL and 0.267 eV at 90% CL. (Note, however, that the actual conclusions above as well as the histograms in Fig. 2 *are valid for the particular situation for which they were done* – i.e. for m_ν=0. For other cases, histograms corresponding to those cases must be constructed and all conclusions must be derived from them.)

6 Energy scale imperfections

There exist various kinds of energy scale imperfections. We concentrate ourselves to two typical ones, namely the wrong calibration and time instabilities. In particular, we are not interested in instabilities of high voltage sources themselves since, during the experiment, the actual voltages are stored together with the count rates and corrections [4] may be done in the spectra evaluation phase. The things we concentrate upon are instabilities of voltmeters and high voltage dividers. These manifest themselves as an energy scale bias and/or improper slope of the calibration line.

6.1 Constant energy bias

Assume that a wrong calibration of our voltmeter leads to an undetected bias, δ, of the energy scale and that this bias does not change during the whole measurement cycle. Let us see how it can affect the resulting (fitted) value of the neutrino mass and its standard deviation. We create the simulated spectrum with the biased energy scale

$$S_{exp}(T_i) = S_{th}(T_i + \delta, N_s, N_b, Q + \delta, m_\nu = 0) \tag{12}$$

and fit it by the "correct" spectrum $S_{th}(T_i, N_s, N_b, Q, 0)$. The results are presented in Table 2. This example was calculated for one year measurement time. Repeating it with various exposure times it appeared that the result, $\overline{m}_\nu$, is independent of it. Then we can conclude that the result is insensitive to a biased voltmeter (assuming the bias does not change

Table 2
Effect of undetected constant energy bias, δ, on the results of the neutrino mass fit. The symbol $\overline{m}_\nu^2$ stands for the fitted result, m_ν^2 (=0 in this case) is the expected result. $\sigma^+_{m_\nu^2}$ is estimation of the resulting upper neutrino mass deviation.

δ [V]	$\overline{m}_\nu^2 - m_\nu^2$ [eV^2]	$\sigma^+_{m_\nu^2}$ [eV^2]
100	-1.02×10^{-10}	0.0424
50	-3.08×10^{-11}	0.0424
30	-1.04×10^{-11}	0.0424
10	-1.10×10^{-12}	0.0424

with time). This is evidently due to the fact that the measured region is quite short and both the spectrum amplitude and the endpoint are fitted parameters. One could mention that the only influence comes from the change of $p(E+m) = \sqrt{E(E+2m)}(E+m)$ in Eq.(1) which is very small.

6.2 Step change of energy bias

Now, let us assume that the voltmeter and voltage divider are well calibrated at the beginning of the measurement but they are not stable, i.e. that may shift during the measurement time. We simplify the situation in the following way: half the time, the calibration is right but the other half, the energy scale shifts (undetected) by δ. Mathematically, it means that we measure

$$S_{exp}(T_i) = \frac{S_{exp,1}(T_i) + S_{exp,2}(T_i)}{2}, \tag{13}$$

where

$$\begin{aligned} S_{exp,1}(T_i) &= S_{th}(T_i, N_s, N_b, Q, m_\nu = 0) \\ S_{exp,2}(T_i) &= S_{th}(T_i + \delta, N_s, N_b, Q, m_\nu = 0). \end{aligned} \tag{14}$$

Again, we fit such measurement by a "correct" spectrum, i.e. the one without the bias. (We used again the case of one year measurement.) In Table 3 we present the fitted neutrino squared masses and the corresponding standard deviations. Also in this case, the resulting mass shifts do not depend on the exposure time. On the other hand, the results are quite sensitive to the value of changing energy scale bias. From Table 3 we see that the energy scale stability of a few ppm (from 18.6 kV) is necessary.

Table 3
Effect of undetected energy bias, δ, on the fitted neutrino mass. The symbol $\overline{m}_\nu^2$ stands for the fitted result, m_ν^2 (=0 in this case) is the expected result. $\sigma^{\pm}_{m_\nu^2}$ are estimations of the resulting upper and lower neutrino mass squared deviations.

δ [V]	$\overline{m}_\nu^2 - m_\nu^2$ [eV^2]	$\sigma^-_{m_\nu^2}$ [eV^2]	$\sigma^+_{m_\nu^2}$ [eV^2]
1.00	-6.427×10^{-1}	0.0669	0.0649
0.50	-1.418×10^{-1}	0.0529	0.0509
0.30	-4.853×10^{-2}	0.0490	0.0460
0.10	-5.155×10^{-3}	0.0462	0.0426
0.05	-1.282×10^{-3}	0.0458	0.0425
0.03	-4.752×10^{-4}	0.0457	0.0424
0.01	-2.500×10^{-5}	0.0457	0.0424

6.3 Gaussian blurred energy bias

In the section 6.2, we have studied sudden change of the energy bias. Here, we consider the case when the instability is not step-wise but somehow "oscillating". To examine such a case, we must expect that the bias during the measurement is of some reasonable probability shape. We utilize here a Gaussian form. So we created the spectrum in the form of

$$S_{exp} = \int_{-\infty}^{+\infty} S_{th}(T_i + u.\delta, N_s, N_b, Q, m_\nu) \times \frac{1}{\sqrt{2\pi}} \exp(\frac{-u^2}{2}) \mathrm{d}u. \tag{15}$$

Such spectra were evaluated using the unbiased theoretical ones. The results for various biases are presented in Table 4. As in the previous cases, the neutrino mass shifts appeared

Table 4
Effect of the Gaussian blurred energy bias. The δ is the expected energy bias, Eq. (15).

δ [V]	$\overline{m}_\nu^2 - m_\nu^2$ [eV^2]	$\sigma^-_{m_\nu^2}$ [eV^2]	$\sigma^+_{m_\nu^2}$ [eV^2]
0.50	-6.228×10^{-1}	0.0656	0.0640
0.30	-2.040×10^{-1}	0.0549	0.0531
0.10	-2.059×10^{-2}	0.0437	0.0434
0.05	-5.027×10^{-3}	0.0462	0.0462
0.03	-1.789×10^{-3}	0.0459	0.0425
0.01	-1.960×10^{-4}	0.0459	0.0424

to be independent on the measurement time.

Since the bias is Gaussian one, the results should fulfill the approximate equality [14]

$$\Delta m_\nu^2 = -2 \times \delta^2. \tag{16}$$

Taking into account the fact that m_ν=0 in Table 4 we see that Eq.(16) holds with reasonable precision.

6.4 Wrong slope of calibration line

Another typical energy scale fault is a wrong slope of the calibration line of the voltmeter. A drifting dividing ratio of a high voltage divider manifests itself as the same effect. Both effects lead to wrong energy scale step, i.e. wrong width of an energy bin.

To study how this affects the fitted neutrino mass, we again created simulated spectra

$$S_{exp} = S_{th}(T_i', N_s, N_b, Q', m_\nu = 0) \tag{17}$$

and fitted them using the correct theoretical spectrum $S_{th}(T_i, N_s, N_b, Q, 0)$. Here

$$T_i' = T_i + (T_i - T_1) \times \delta, \qquad Q' = Q + (Q - T_1) \times \delta \tag{18}$$

and δ is the relative change of the energy step. We expect δ *unvarying* all the time. Results are in Table 5. As well as in all the cases studied above, the fitted neutrino mass shifts

Table 5
Effect of wrong slope of the calibration line. The δ is the relative change of the energy step.

δ	$\overline{m}_\nu^2 - m_\nu^2$ [eV^2]	$\sigma^+_{m_\nu^2}$ [eV^2]
1×10^{-3}	-1.073×10^{-2}	0.0431
5×10^{-4}	-5.314×10^{-3}	0.0427
1×10^{-4}	-1.050×10^{-3}	0.0425
5×10^{-5}	-5.290×10^{-4}	0.0424
1×10^{-5}	-1.040×10^{-4}	0.0424
5×10^{-6}	-5.184×10^{-5}	0.0424
1×10^{-6}	-9.610×10^{-6}	0.0424

are independent on the measurement time.

6.5 Varying slope of calibration line

The last imperfection we have examined is a time-dependent modification of the previous one. This again may include time instability of both the voltmeter and the high voltage divider. Such an imperfection may exhibit itself by various manners. For our study we chose the following simple one.

Let us create two simulated spectra: a "correct" one and the second one with changed energy step. Let both of them have identical endpoint. Then we add the two spectra together and fit the resulting spectrum with a "correct" theoretical one. (It corresponds to a situation when we measure part of the time with the right energy step and part of the time with a changed one.) The results are presented in Table 6. As well as in all the previous cases, no dependence of the neutrino mass shifts on the measurement time was observed.

Table 6
Effect of the varying slope of the calibration line. The δ is relative change of the energy step.

δ	$\overline{m}_\nu^2 - m_\nu^2$ [eV^2]	$\sigma^-_{m_\nu^2}$ [eV^2]	$\sigma^+_{m_\nu^2}$ [eV^2]
1×10^{-3}	-5.806×10^{-3}	0.0463	0.0427
5×10^{-4}	-2.746×10^{-3}	0.0460	0.0425
1×10^{-4}	-5.236×10^{-4}	0.0457	0.0424
5×10^{-5}	-2.723×10^{-4}	0.0457	0.0424
1×10^{-5}	-5.476×10^{-5}	0.0457	0.0424
5×10^{-6}	-2.916×10^{-5}	0.0457	0.0424
1×10^{-6}	-5.290×10^{-6}	0.0457	0.0424

7 Conclusion

In this work we have investigated how small undetected energy scale imperfections may affect neutrino mass resulting from an electron spectroscopy experiment. All the studies were done for the conditions very close to the ones expected in the first stages of the KATRIN experiment design [1].

We have developed a numerical model of the experiment. It is based on integrated β-spectra derived from differential spectra corresponding to four-fermion static approximation and the theoretical instrumental transmission function. Corrections for electron energy losses due to inelastic scattering in the source as well as for rotational-vibrational excitation states of the daughter $(^3\mathrm{HeT})^+$ molecule were considered.

The spectra were evaluated using the least squares method with four free parameters

(spectrum amplitude, background, endpoint, and neutrino mass). The fitted parameter deviations were determined by two independent methods. The algorithms developed enabled us to fit neutrino mass or neutrino mass squared.

The accuracy and consistency of the model were found to be satisfactory. It was proved that the physical predictions do not depend on whether linear or squared neutrino mass was fitted. The standard deviation of the neutrino mass squared was shown to be practically independent of the neutrino mass. The neutrino mass histograms enabled us to construct the neutrino mass probability density function and thus to connect the standard deviation with appropriate confidence levels.

The influence of possible undetected energy scale imperfections was examined in detail. We demonstrated that the results are completely insensitive to constant energy scale bias. On the other hand, an unstable bias produces an effect which must be carefully minimized. The study of faulty slope of the calibration line showed, on the contrary, that static variant of this imperfection leads to higher error than the dynamic one.

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Figure caption

Fig. 1. Shape of the response function of the KATRIN spectrometer.

Fig. 2. Histograms evaluated to derive the neutrino mass cumulative distribution function. Both cases linear (a) and squared (b) neutrino mass are presented. For details see text.